Reduction of Flow-Measurement Uncertainties in Laser Velocimeters with Nonorthogonal Channels

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An analysis of certain geometrical limitations inherent in the application of laser velocimeters with nonorthogonal channels has led to the development of advanced LDA-calibration and data-acquisition techniques that minimize systematic and statistical errors, respectively. The data-acquisition technique optimizes the number of velocity samples collected from three velocimeter channels as a function of local turbulence intensity, vector direction, and prescribed confidence interval. Linear velocity surveys and streamline traces measured in a turbulent flowfield with a three-dimensional laser velocimeter are presented and the validity and accuracy of the theoretical analysis are discussed.

Nomenclature

A	$= 3 \times 3$ matrix with elements a_{ij}
a_{ij}	= elements of matrix A
f_i	= frequency measured from laser Doppler anemometer (LDA) channel i
N	= ensemble size
s_i	= standard deviation of distribution of frequencies measured from LDA channel i
s_F	= standard deviation of distribution of a generalized quantity F
U, V, W	= orthogonal velocity components parallel to x, y, and z axes, respectively
V_i	= velocity measured from LDA channel i
V_R	= three-dimensional resultant velocity
$V_{R\theta}, V_{R\gamma}, V_{R\alpha}$	= projection of three-dimensional resultant velocity in y-z, x-y, and x-z planes, respectively
x, y, z	= coordinate axes
z_c	= confidence coefficient
α	= velocity-vector projection angle in x-z plane; positive from z axis
β_i	= LDA beam pair half angle for channel i
γ	= velocity-vector projection angle in x-y plane; positive from x axis
θ	= velocity-vector projection angle in y-z plane; positive from z axis
λ_i	= wavelength of LDA channel i
σ / V_R	= isotropic turbulence intensity
ϕ	= coupling angle; angle between two nonorthogonal channels
Δ	= error or change in quantity that follows
$\overline{(\ )}$	= mean or average of quantity

Introduction

SINCE the introduction of the laser Doppler anemometer (LDA), several schemes have been suggested for simultaneous three-channel measurement and determination of the three orthogonal components of local velocity. An early three-dimensional (3D) local-oscillator system by Fridman et al.[1] used three nearly on-axis detectors wherein the three channels were coupled by small angles; hence, accurate resolution of the transverse velocity components was difficult to obtain. Yanta[2] has reported measurements with a forward-scatter 3D LDA employing a third dual-scatter channel oriented at a 30-deg angle to a conventional two-color, two-channel coaxial LDA. Abbiss et al.[3] have reported measurements with a similar system that operates in backscatter. TSI, Inc.[4] has recently reported the development of a 3D LDA using a five-beam output pattern through a common lens resulting in small coupling angles between the channels that are used to extract the on-axis velocity component. The measurements described in the present paper were made with an improved version of a 3D backscatter LDA previously reported by Snyder et al.[5] The instrument incorporates a third, off-axis (nominally 17 deg) dual-scatter channel, and the third orthogonal velocity component is resolved from a combination of this channel and one other channel.

A common feature in all these 3D LDA instruments is a geometrical arrangement that requires the determination of at least one orthogonal velocity component from a transformation equation involving at least two nonorthogonally sensed velocities. Orloff and Snyder[6] have noted that such arrangements are highly susceptible to calibration and sampling errors that may cause erroneous measurement of the third (coupled) velocity component. Their results indicate that small errors in the conversion constants (frequency-to-velocity) for a particular LDA instrument can result in large errors in the third component and that the extent of these errors depends on the magnitude and the direction of the local velocity vector (in the plane of the coupled components), the local turbulence intensity, and the number of velocity samples that comprise the data ensemble.

The present paper generalizes the results of Orloff and Snyder so that any 3D LDA may be described in terms of a transformation matrix, the elements of which contain all the pertinent geometrical characteristics of the instrument that are necessary to convert three measured Doppler frequencies to three orthogonal velocity components. A similar matrix representation has been reported by Crosswy,[7] who has considered the influence of the matrix elements on the systematic errors only. This concept will be further developed in the present paper to show that 1) systematic errors can be reduced by using an improved method for determining the elements of the transformation matrix, 2) the transformation matrix provides a logical way of estimating the statistical errors in the velocity components, and 3) statistical errors can be reduced to any desired level by adjusting the size of the data ensemble to account for changes in the local flow conditions.

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The theory and techniques to be presented herein are important in the application of 3D LDA instruments to the measurement of complex turbulent flows. In many experiments, flow patterns are studied by means of vector plots that are useful in resolving mean streamline patterns. Investigations of this type have already been conducted by many researchers applying the LDA to a variety of complex flows. For example, Walker et al.[8] used a two-component LDA to determine flow patterns in the intrablade region of a transonic fan; Young et al.[9] have used a two-component LDA to study the separated flow over a stalled wing; and Martin et al.[10] have used an LDA to determine the vector distributions in a model heat exchanger.

The theory to be presented suggests that where vector plots are desired the ensemble size should be varied so as to maintain constant vector directional accuracy even though the flow conditions may change significantly along the LDA survey line. Examples of constant-accuracy surveys will be presented to verify this theory.

The theoretical concepts developed in this report have been applied to streamline tracing, and a preliminary study has been conducted on the feasibility of the technique. Unfortunately, reconstruction of mean streamline patterns from straight-line surveys requires large amounts of data to be stored and later sorted, reduced, and, finally, presented as vector plots. Even then, any particular streamline can only be inferred from the vector mapping. A better method would be to place the LDA test point at some location in the flow, accurately measure the direction of the 3D velocity vector, move a short distance in that direction (assuming three-axis test point positioning is available), and repeat the procedure at each new location, thereby tracing out the mean streamline itself.

The streamline-tracing technique is particularly attractive where high levels of turbulence are present, flow visualization is impossible, and information is needed regarding fluid dynamic processes such as mixing and entrainment. Also, the LDA is an ideal instrument for applications where high levels of turbulence are accompanied by regions of reversed and recirculating flow. An example is presented of a streamline trace conducted in a nominally 50%-turbulent highly 3D flow, and the limitations of the technique are discussed.

LDA Calibration

Calibrations of an LDA where two or more channels are nonorthogonal is a critical step in insuring that the instrument will provided mean velocity measurements with low systematic uncertainties. Conventional means of calibrating LDA systems with orthogonal channels often entail making detailed measurements of the laser-beam geometry to determine the exact conversion constants relating frequency of fringe crossings to particle velocity. In Ref. 6 it is shown that although geometrical measurements may be adequate enough to determine conversion constants for single-component LDA systems, a multiple-channel system with nonorthogonal components will suffer because of the way in which the errors in the constants compound and affect the resulting computed orthogonal velocity components.

Figure 1 shows a simple arrangement of two LDA channels where channel 1 is directly sensing the W-component of the in-plane velocity projection V_R. Channel 2 is arranged nonorthogonal to channel 1 with a coupling angle of ϕ. In this configuration, the coupled component V is resolved from velocity information obtained from LDA channels 1 and 2. Mathematically, this relationship may be expressed as

$$V = (V_2/\sin\phi) - (V_1/\tan\phi) \tag{1}$$

where V_1 and V_2 are velocities measured independently by each LDA channel and are computed from the standard dual-scatter LDA equations

$$V_1 = \lambda_1 f_1 / 2\sin\beta_1, \qquad V_2 = \lambda_2 f_2 / 2\sin\beta_2 \tag{2}$$

Inspection of Fig. 1 reveals that, in general, V_1 and V_2 will be nearly equal in magnitude. In fact, as the coupling angle ϕ is decreased, V_1 and V_2 remain close in magnitude over larger ranges of the flow angle θ. Accordingly, $\sin\phi$ and $\tan\phi$ will be small, thereby increasing the value of each term in Eq. (1). Therefore, the coupled component V must be resolved as the difference of two large numbers of nearly the same magnitude. Unfortunately, $\sin\beta_1$ and $\sin\beta_2$, the geometrical parameters for the LDA channels 1 and 2, respectively, also appear in the denominator of each term in Eq. (1). As a result, because β_1 and β_2 are small, the terms become even larger in magnitude. The overall effect is that small relative uncertainties in β_1, β_2 and ϕ can cause large relative uncertainty in V, especially for smaller values of V.

The analysis presented in Ref. 6 for a nonorthogonal LDA system with a coupling angle ϕ of 14 deg shows that a calibration performed by making length measurements of the beam geometry can only insure a systematic accuracy in V/V_R of between 2 and 16%, depending on the flow direction θ.

An alternate method of calibrating a nonorthogonal LDA might involve the use of a reference instrument that would provide a velocity of known magnitude and direction. An accurate reference velocity would allow one to relate directly measurements of frequency obtained from the LDA channels to the actual velocities being measured. Higher confidence and, thus, lower systematic uncertainties could be realized using a technique of this type.

The authors have constructed a reference instrument capable of providing a velocity within the LDA focal volume with a magnitude of 19.15 m/s to an accuracy of 0.1%, and an adjustable direction that is accurate to 0.1 deg relative to any selected frame of reference. Although the details of this device will not be discussed here, it is worth considering the concept behind this instrument.

Since measurements with an LDA are made by observing the movement of particles suspended in the flow, a reasonable velocity reference instrument would directly simulate the motion of such particles. One possible method involves a very fine wire whose diameter is approximately that of seeding particle (5μm); an alternate method would be to replace the wire with a fine knife edge. The wire or knife edge could be translated through the measuring volume of the LDA by attaching it to a rotating disk. By controlling the speed of the motor and knowing the precise radius at which the wire intersects the measuring volume, and accurate velocity magnitude can be obtained. Using an arrangement of rotation and translation stages, the disk can be oriented in a variety of positions so that different velocity directions can be selected.

Details of the geometry of the LDA such as β_1, β_2, and ϕ in Eqs. (1) and (2) are unnecessary when LDA calibration is

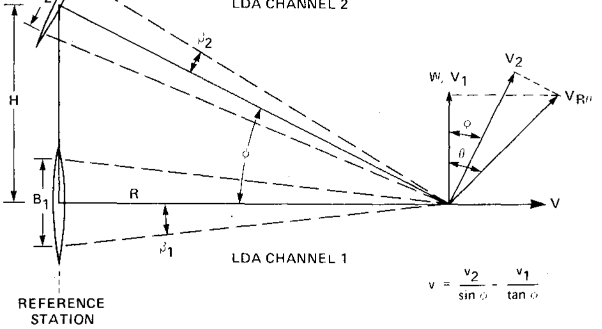


Fig. 1 Nonorthogonal LDA channels and velocity-component definitions.

done using an accurate velocity reference. In fact, this technique is applicable in general for any arbitrary 3D LDA geometry that may incorporate more than two nonorthogonal channels within the optical system. In the general case, it is convenient to define a simple linear relationship between the measured 3D LDA frequencies f_1, f_2, and f_3 and the computed orthogonal velocities U, V, and W, relative to any particular frame of reference. For any fixed-geometry LDA system, the relationship is given by the matrix expression

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & {}_{33} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \tag{3}$$

where the calibration matrix A with elements a_{ij} converts frequency information to orthogonal velocity components.

At this point, the calibration is simplified to the process of determining the matrix A by using a velocity reference to generate U, V, and W, and then measuring f_1, f_2, and f_3 directly through the LDA data-acquisition equipment. A set of nine known velocity components and nine measured frequencies are sufficient to explicitly solve for the elements a_{ij}.

The authors have applied this calibration technique to an optical scanning 3D LDA with a single nonorthogonal component as indicated in Fig. 1. Because the LDA has scanning optics, the geometry of the output beam arrangement changes as a function of scan range. This means, of course, that there is a unique matrix for every range location and the calibration process, therefore, is complicated by introducing curve fits and interpolation tables into the LDA data-acquisition scheme. In spite of the added complications generated by a scanning LDA, it was possible to achieve a worst-case measurement uncertainty ($\Delta V/VR$) in the critical component V of less than 2% as compared with 16% previously obtained with geometrical measurements. It is estimated that for a fixed-focus LDA system, the overall accuracy could be maintained to well within a 1% error margin.

Statistical Considerations—Estimation and Sampling Theory

While systematic errors can be minimized through a specialized calibration technique, statistical errors are generated from random phenomena occurring in the flowfield as well as in the measurement process and must be handled with estimation and sampling theory. Once the statistical errors have been expressed mathematically, it is possible to determine, for a given confidence level, how many samples of data are necessary to obtain a prescribed level of accuracy in the measured velocity. It has been shown[11] that, in general, for a function

$$F=f(x_1, x_2, x_3,\ldots) \tag{4}$$

where x_i are the independent variables, the statistical quantities $\bar{F}$ and $s_{\bar{F}}^2$ are described by

$$\bar{F}=f(\bar{x}_1, \bar{x}_2, \bar{x}_3,\ldots) \tag{5}$$

and

$$s_F^2=\left(\frac{\partial F}{\partial x_1}\right)^2 s_1^2+\left(\frac{\partial F}{\partial x_2}\right)^2 s_2^2+\left(\frac{\partial F}{\partial x_3}\right)^2 s_3^2+\ldots \tag{6}$$

provided that the x_i variables are random and independent samplings and the s_i^2 quantities are the best estimates for the variances of the distributions of the x_i samplings. Of particular interest is the standard deviation of the distribution of mean values $\bar{F}$ expressed as

$$s_{\bar{F}}=s_F/(N)^{1/2} \tag{7}$$

from which a confidence interval for the mean value $\bar{F}$ can be written by including a confidence coefficient z_c for normal distributions. For example, one can be 95% confident that a computed mean value $\bar{F}$ will not be in error from the true mean by more than $\pm\Delta F$ when ΔF is expressed as

$$\Delta F=1.96\, s_F/(N)^{1/2} \tag{8}$$

Again, it is convenient to use the matrix formulation given by Eq. (3) to develop equations describing the statistics in terms of the system geometry as implied in the a_{ij} matrix elements. From Eq. (3), U can be written as

$$U=a_{11}f_1+a_{12}f_2+a_{13}f_3 \tag{9}$$

Note that for Eq. (6) to be compatible with Eq. (9), the samples of frequencies f_1, f_2, and f_3 must be independent of one another; that is, it is assumed that data are acquired in such a manner that no correlation exists between the frequency channels. Reference 6 discusses this case as well as the effect of correlated (simultaneous) sampling of the velocity that results in statistical uncertainties lower than those resulting from correlated data. The case in which the data are correlated is reserved for future study.

Application of Eqs. (4) and (6) to Eq. (9) yields

$$s_U^2=(a_{11}s_1)^2+(a_{12}s_2)^2+(a_{13}s_3)^2 \tag{10}$$

where s_1^2, s_2^2, and s_3^2 are variances in the frequency ensembles from channels 1, 2, and 3, respectively, of the LDA. Similar equations may be derived for the V and W velocity components. A concise expression for all three variances of the orthogonal velocities is

$$\begin{bmatrix} s_U^2 \\ s_V^2 \\ s_W^2 \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{12}^2 & a_{13}^2 \\ a_{21}^2 & a_{22}^2 & a_{23}^2 \\ a_{31}^2 & a_{32}^2 & a_{33}^2 \end{bmatrix} \begin{bmatrix} s_1^2 \\ s_2^2 \\ s_3^2 \end{bmatrix} \tag{11}$$

Uncertainties in the mean values $\bar{U}$, $\bar{V}$, and $\bar{W}$ may now be expressed from Eq. (8) as

$$\Delta U=1.96\, s_U/(N)^{1/2}$$

$$\Delta V=1.96\, s_V/(N)^{1/2}$$

$$\Delta W=1.96\, s_W/(N)^{1/2} \tag{12}$$

Equations (11) an (12) offer a workable means for relating the number of samples in the ensemble and the ensemble standard deviations to the estimated errors in $\bar{U}$, $\bar{V}$, and $\bar{W}$ for a three-channel LDA of arbitrary geometry.

As indicated earlier, many LDA experiments involve studying a flow by means of vector plots to determine the streamline character of the flow. In this case, it is more appropriate to apply the statistical analysis to the estimated error $\Delta\theta$ in the angle θ that an in-plane vector V_R makes with the vertical axis, as illustrated in Fig. 2. The more general case of three dimensions involves vector projections in the x-y, y-z, and z-x planes; then, the directional accuracy of the velocity vector is specified by considering the estimated errors in the directions of the projected vectors.

The angle θ of the vector projection in the y-z plane (see Fig. 2) can be written in terms of frequencies f_1, f_2, and f_3 and the elements of the matrix A. After expressing this geometrical relationship in the form of Eq. (4),

$$\theta = f(f_1, f_2, f_3) \tag{13}$$

an application of Eq. (6) yields an expression for the variance of the distribution of θ,

$$s_\theta^2 = \frac{1}{V_{R\theta}^2} \sum_{j=1}^{3} (a_{2j}\cos\theta - a_{3j}\sin\theta)^2 s_j^2 \tag{14}$$

Similar expressions follow for other coordinate planes. For the x-y plane where γ is measured from the x axis,

$$s_\gamma^2 = \frac{1}{V_{R\gamma}^2} \sum_{j=1}^{3} (a_{2j}\cos\gamma - a_{1j}\sin\gamma)^2 s_j^2 \tag{15}$$

For the x-z plane where α is measured from the z axis,

$$s_\alpha^2 = \frac{1}{V_{R\alpha}^2} \sum_{j=1}^{3} (a_{1j}\cos\alpha - a_{3j}\sin\alpha)^2 s_j^2 \tag{16}$$

In each of these applications, $V_{R\theta}$, $V_{R\gamma}$, and $V_{R\alpha}$ are the in-plane projections of the velocity vector.

Similar to the analysis given for the orthogonal components U, V, and W, the 95% confidence limits for θ, γ, and α are computed using Eq. (8),

$$\begin{aligned} &y\text{-}z \text{ plane} \quad && \Delta\theta = 1.96\, s_\theta / (N)^{1/2} \\ &x\text{-}y \text{ plane} && \Delta\gamma = 1.96\, s_\gamma / (N)^{1/2} \\ &x\text{-}z \text{ plane} && \Delta\alpha = 1.96\, s_\alpha / (N)^{1/2} \end{aligned} \tag{17}$$

Note that statistical uncertainty equations may be derived for other quantities of interest, such as the projected vector lengths $V_{R\theta}$, $V_{R\gamma}$, and $V_{R\alpha}$; the absolute vector magnitude; or any other magnitudes or directions that are of interest to the experimenter.

Verification of Statistical Uncertainty Equations

Examination of the statistical uncertainty Eqs. (12) and (17) reveals that there are three major factors influencing the predicted errors in the LDA measurement of the vector velocity at any given location within the flow: 1) the geometry of the LDA system, as contained in the values of the matrix elements a_{ij}; 2) local turbulence in the flow, or, for that matter, any other factor that results in statistical broadening of the distribution of frequency measurements, represented statistically as s_1^2, s_2^2, and s_3^2; and 3) the inverse square root of the ensemble size N. Additionally, as the test point is moved to other locations within the flow where local conditions may be different, Eqs. (14-16) suggest that the flow direction itself (defined by θ, γ, and α) will strongly influence the uncertainties.

To verify that the estimated errors predicted by the statistical theory are correct, a special LDA experiment was conducted. The experiment used a seeded air jet of stable magnitude and direction; the jet was positioned to pass through the measurement volume of a nonorthogonal LDA. The measured distribution of θ and V were plotted as a function of ensemble size N for different turbulence intensities. The corresponding theoretical confidence intervals from Eqs. (12) and (17) were then compared to the measured distributions. In this manner, the validity of the statistical equations could be evaluated.

A three-channel LDA was used to perform the experiment; two of the channels are nonorthogonal and are arranged as shown in Fig. 1. One channel measures the W component of velocity directly; the other channel is coupled by 15.5 deg so that the coupled component V can be resolved. The transformation matrix elements a_{ij} were computed by calibrating in the manner discussed in the previous section.

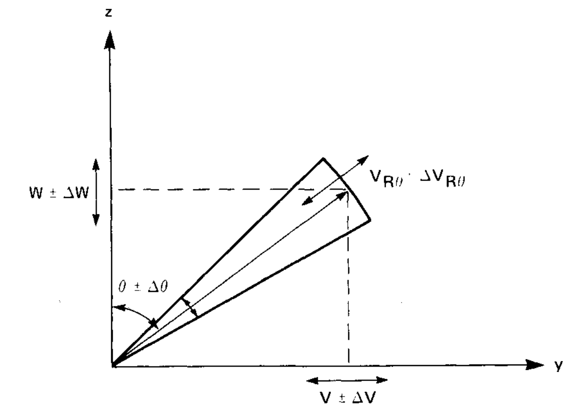


Fig. 2 Velocity-component uncertainties and directional uncertainty.

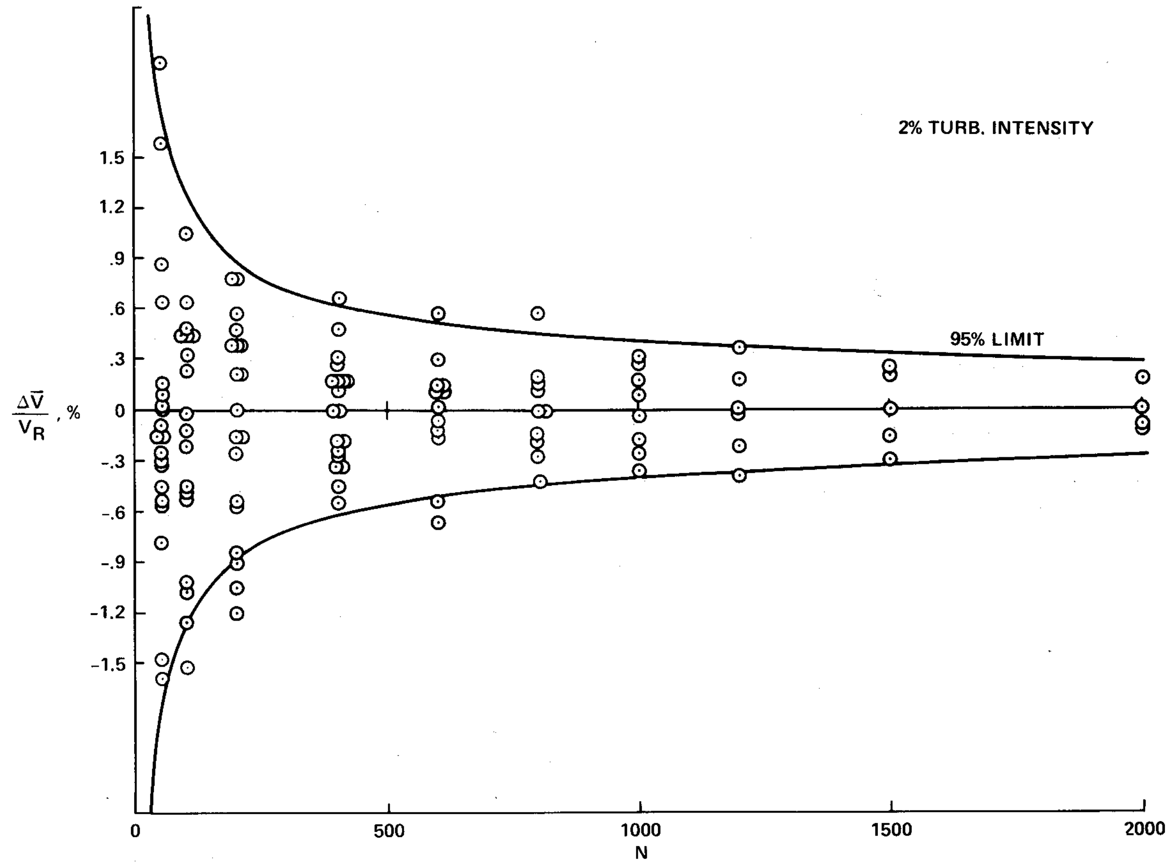
A small (6-mm-diam) air jet was introduced at the LDA measuring volume at an arbitrary angle with respect to the LDA, and 10,000 samples of data were acquired from each of the three LDA channels. This was done for a case of low-turbulence (2%) and a case of high-turbulence (27%) intensity. The frequency samples were reduced in the following manner. First, the mean velocities and angles were computed using all 10,000 frequency samples, and the results were taken as representative of the true population means. Next, for a given ensemble size N (much less than 10,000), mean values were computed by randomly selecting N samples from the set of 10,000; care was taken not to reuse any of the previously used N samples to avoid biasing the distribution of mean values. Finally, distributions of $\bar{V}/V_R$ and θ (V_R being the 3D resultant velocity) about the population means were plotted as $\Delta\bar{V}/V_R$ vs N and $\Delta\theta$ vs N for the low- and high-turbulence conditions. The results are presented in Figs. 3 and 4.

The solid curves in Figs. 3 and 4 represent the 95% confidence limits as computed from Eqs. (12) and (17). Ideally, 95% of the experimental points should lie within the curve boundaries. The actual percentage is computed to be not less than 90% and not more than 96%, depending on how one chooses to treat points that are very near the boundary. (It is not unreasonable to assume a small uncertainty in the location of the individual mean values used to generate Figs. 3 and 4 as a result of small errors in the population means incurred using only a finite sample of 10,000.) The agreement between the theory and the experiment is encouraging and suggests that the statistical formalism that has been developed provides a sound basis for further application of the 3D LDA to a complex, highly turbulent flow, as will be described in the following sections.

Statistics and Data Acquisition

The previous sections have concentrated on how to minimize the systematic uncertainties and on understanding the parameters that affect the statistical uncertainties inherent in a nonorthogonal LDA system. Now it is appropriate to apply the theory of statistical uncertainties to 3D LDA measurement of turbulent, three-dimensional flow, using a real-time data-acquisition technique.

Once the calibration of a nonorthogonal LDA system has been performed, the systematic errors are presumably reduced to acceptable levels and remain constant during the application of the LDA to the measurement of an actual fluid flow. However, statistical error will vary with location in the flow. During data acquisition, it is desirable to have a real-time knowledge of the statistical errors. In practice, it is



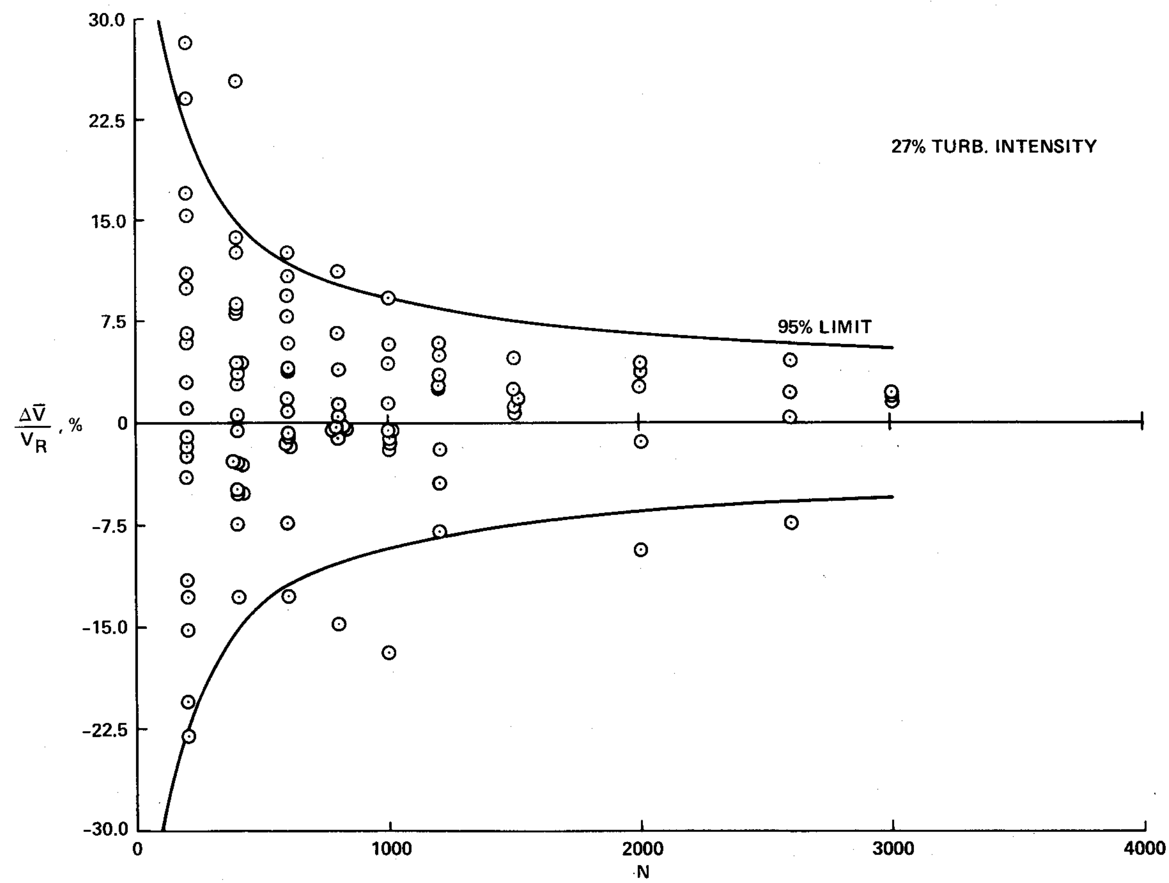


Fig. 3 Relative statistical variation in the mean velocity $\Delta\bar{V}$ about the population mean as a function of sample size N for cases of low- and high-turbulence intensity. Curves show 95% confidence level predicted from Eq. (12).

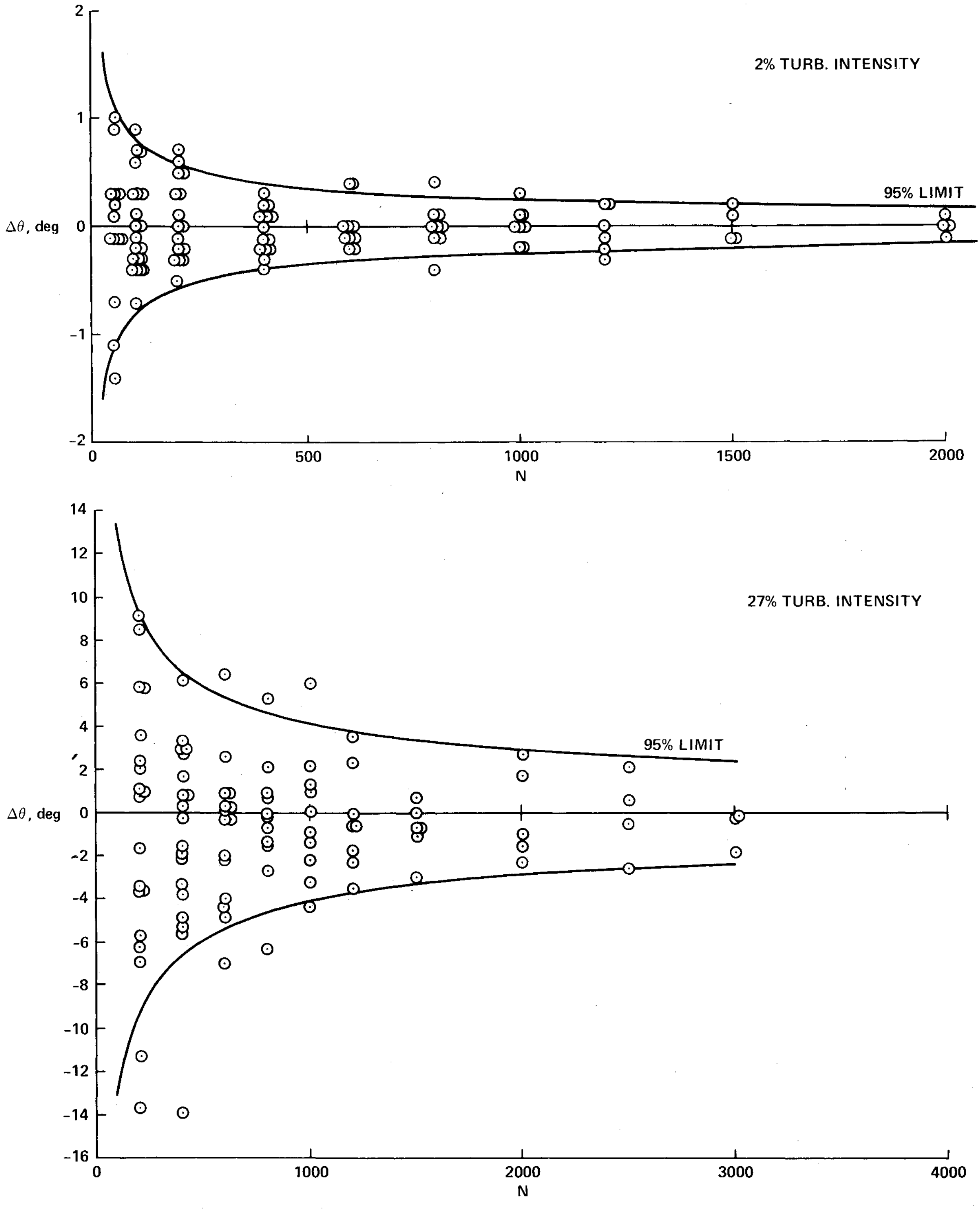


Fig. 4 Relative statistical variation in the flow angle $\Delta\theta$ about the actual direction as a function of sample size N for cases of low- and high-turbulence intensity. Curves show 95% confidence level predicted from Eq. (17).

convenient to specify a tolerable error and then compute the minimum ensemble size N that is required to obtain this accuracy. To accomplish this, a rough estimation of the turbulence intensity and the flow direction and magnitude may be made using a small number of samples. Using these approximate values, it is possible to obtain a more accurate estimate of the ensemble size required to maintain the flow measurements within the specified tolerance. In this manner, "constant-error" data acquisition is possible under software control.

As the LDA test point is moved along a survey line in a complex flow in which the turbulence intensity may vary significantly, the number of data samples required to maintain a given accuracy will, likewise, vary over a large range. For example, for a nonorthogonal LDA with a 15.5-deg coupling angle (Fig. 3), the critical component V requires only 150 velocity samples in a low-turbulence flow to obtain accuracy within a 1% error margin in $\bar{V}$, whereas several thousand samples may be required in a high-turbulence flow. If, on the other hand, the flow *direction* is more important than the velocity *magnitude*, and higher uncertainties in critical components such as V are acceptable, then for a 5-deg uncertainty (Fig. 4), the number of samples required will vary from just a few samples in low turbulence to 700 samples in a

27% turbulence flow.

Equations (14-17) indicate that, in general, fewer samples are required to obtain accuracy in the flow direction than in the vector magnitude. However, there are certain flow directions that will require larger numbers of samples in order to maintain a given accuracy. It has previously been shown[6] that the flow direction perpendicular to the bisector of the two coupled LDA channels requires the greatest number of samples to resolve V to any given accuracy, and the fewest number of samples are required to resolve a flow direction parallel to the bisector of the coupled LDA channels. The equations also indicate that larger numbers of samples are required when the magnitude of the velocity vector becomes very small. As a result, there may be regions of a flow where it is necessary to limit the ensemble size to a maximum value that is compatible with the limitations imposed by the attainable LDA data rates and the data-reduction speed.

It should be emphasized that, if one chooses the error in the flow direction rather than the error in orthogonal velocity components as the criterion for determining the ensemble size N, then Eqs. (14-17) would require that difficult components such as V need only have sufficient accuracy to give a flow direction within the specified error. Hence, data-acquisition time is used optimally for the purpose of determining flow direction with a compromise in the accuracy of the velocity components themselves.

Vector Plots

A simple flow experiment was set up in the laboratory to demonstrate the technique of making constant error measurements of flow direction. A small axial fan was mounted near the exit of a 7.6-cm-diam duct (Fig. 5). This highly turbulent swirling flow provided an interesting case for evaluation of the data-acquisition technique. A 3D LDA system with two orthogonal channels and a single nonorthogonal channel was used to make a radial velocity survey (parallel to the z axis) at a location 0.2 cm from the duct exit. The LDA beam configuration (Fig. 5) was oriented such that the coupled component V would be representative of the swirl component in the flow. Axial and radial velocities were measured directly by two orthogonal dual-scatter LDA channels parallel to the x and z axes, respectively.

Figure 6 shows the results of two radial velocity surveys with different directional error tolerances. To clearly illustrate flow direction, the data are presented as vector plots so that subtleties in flow direction can be noted. The measured vectors in the V-W plane provide an end view from which one can visualize the swirl structure of the flow. The vectors in the U-W plane give a profile view showing higher velocity air concentrated at the edge of the duct and reverse flow at the center.

In Figs. 6a and 6b, the vector directional error in the V-W plane is limited to 30 deg and 5 deg, respectively, in order to demonstrate how the directional error influences the character of a vector plot. As expected, a significant amount of directional scattering is evident in the 30-deg case, with an

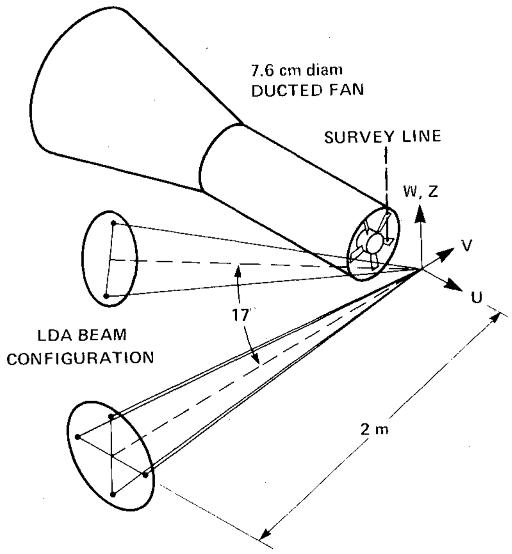


Fig. 5 Sketch of ducted-fan setup used to generate a turbulent flow, showing orientation of 3D LDA beams with respect to flow.

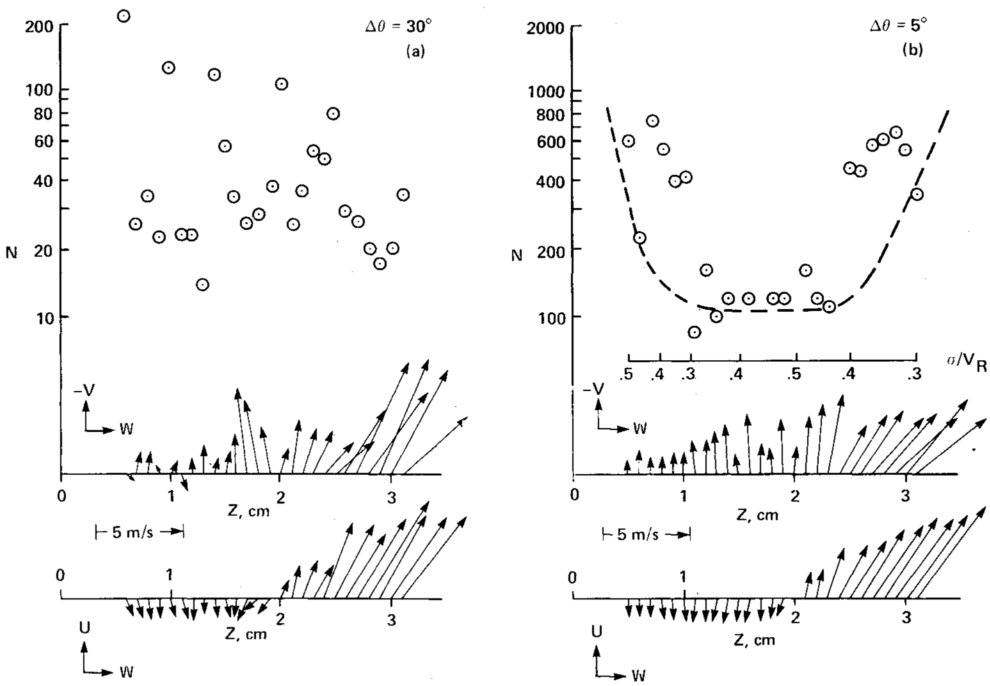


Fig. 6 Radial survey of the turbulent ducted-fan flow showing improvement in directional continuity of vectors as error tolerance is changed from a) 30 deg to b) 5 deg. Number of samples N is determined from statistical theory to give 95% confidence that the V-W plane directional error is less than the prescribed tolerance.

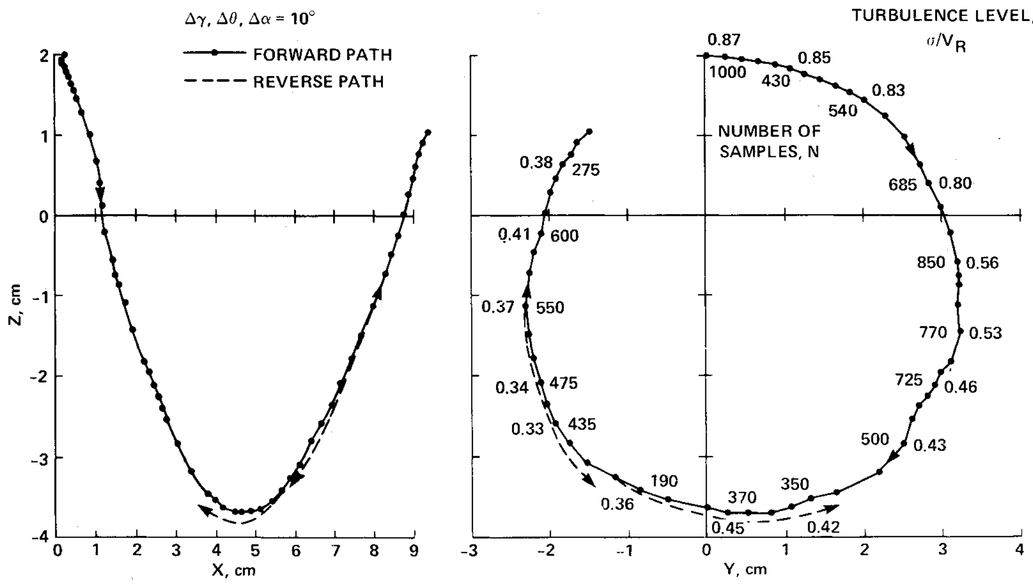


Fig. 7 Streamline trace for 95% confidence that the directional error in all planes is less than 10 deg. Note the variation in the number of samples required as vector direction and turbulence level change. Also, the reverse path indicates cumulative errors in following a true fluid streamline.

occasional vector pointing in the reverse direction; large variations in the vector magnitudes are also evident. On the other hand, a definite improvement in the vector directions is apparent when the estimated directional error is limited to 5 deg, with a similar improvement in the continuity of the vector magnitudes.

The number of samples N (collected in each LDA channel) was adjusted to limit the directional error in the V-W plane only. Because U and W are determined from orthogonal LDA measurements, fewer samples are required to obtain equivalent directional accuracy for vectors in the U-W plane. Hence, because the same number of samples were taken in each channel, the directional error for the U-W plane vectors shown in Fig. 6 is somewhat less than 30 deg or 5 deg, respectively.

Figure 6 shows a definite improvement in the vector plots in both the V-W and U-W planes when changing from 30-deg to 5-deg directional error. Notice that the point of axial flow reversal in the U-W plane becomes more sharply defined and the magnitudes become more continuous as the accuracy criterion is increased.

Ensemble size N and turbulence intensity σ/V_R are also presented in Fig. 6. In the 30-deg case, fewer samples were required (less that 200), in general, with no discernible trend in the variation of N with radius. On the other hand, the 5-deg case required higher numbers of samples near both ends of the survey line. These observations are in agreement with the general results of the statistical theory. As the vector magnitude becomes small, greater numbers of samples are required to define the flow direction accurately; in fact, as the vector magnitude goes to zero, an infinite number of samples are required. Near the outside of the flow ($Z \approx 3$ cm) the magnitudes are not small and, thus, have no significant effect on ensemble size. Instead, it is the change of flow direction in the V-W plane that results in an increased number of samples being needed to resolve the flow direction to the prescribed accuracy.

Streamline Tracing

The most fundamental approach to streamline tracing with a 3D LDA is to measure the three components of the velocity at a given location, compute the flow direction (θ, γ, and α), move a short distance in the direction of the velocity vector, and repeat the procedure. In this manner, the motion of the LDA test point should trace out the mean streamline, even where turbulence levels are high. For streamline tracing, overall accuracy in the values of $\bar{U}$, $\bar{V}$, and $\bar{W}$ are not as important as accurate measurements of θ, γ, and α; therefore, in regions of the flow where $\bar{V}$ is large relative to $\bar{W}$, only a limited number of data samples are required to compute the flow direction to a prescribed accuracy in the plane of the coupled components.

Figure 7 shows results of streamline tracing in the ducted-fan flow described earlier. The estimated directional error was held constant at 10 deg for all measurements. The starting point is located 0.2 cm beyond the duct exit. Each dot along the trace represents the position of the LDA test point where the velocity was measured before stepping to the next location.

A step-size algorithm was included in the data acquisition and motion software to reduce the step size where the radius of curvature of the streamline is small and increase the step size where the curvature is large. Notice that the number of samples required for 10-deg accuracy varies over a wide range as the vector direction and the turbulence intensity vary. Considering the levels of local turbulence in this flow, it is encouraging that a reasonably smooth trace was obtained. In fact, the continuity and smoothness of the trace are further verification of the statistical theory.

Unfortunately, the forward path shown in Fig. 7 is not a faithful representation of a streamline. This is not a fault of the LDA, the data-acquisition system, or the statistical theory. Instead, the problem lies in the fact that the step segments are always straight lines tangent to the local streamline. Clearly, for any amount of streamline curvature the error introduced by a straight-line step is not large, but the cumulative effect is significant. To verify this effect, two reverse-path segments (moving opposite to the velocity direction) are shown in Fig. 7. The divergence to the outside of the forward trace is to be expected based on the straight-line stepping procedure.

Concluding Remarks

Progress in laser Doppler anemometry has advanced to the point where fully three-dimensional optical systems are becoming more common, and the fluid dynamicist's attention can be directed more toward LDA applications than development. Because an LDA can sense velocity without directional ambiguity, stagnation and recirculation regions pose no particular measurement difficulty, and one can seriously consider the application of 3D LDA instruments to complex flows with high-turbulence intensity. Heretofore, diagnosis of such flows have been a problem. Flow visualization is nearly impossible due to turbulent mixing and high mean shear, and other velocity-measuring devices are typically not capable of handling regions of flow reversal, especially when 3D measurements are involved.

However, the sampling procedure in LDA data acquisition, and the complications brought about by 3D LDA optical systems with nonorthogonal channels, have necessitated a reassessment of the manner in which the orthogonal components of the mean velocity vector are determined. Without improved data-acquisition and reduction methods that maintain constant accuracy in difficult flow regions (e.g., by varying sample size N), the systematic and statistical errors may inadvertently exceed acceptable levels in these regions.

It has been shown that a 3D LDA of arbitrary optical geometry can be concisely represented by a transformation matrix whose elements contain all pertinent parameters of the instrument. In general, systematic errors in a velocity measurement are related directly to the precision with which the matrix elements are determined; where nonorthogonal channels are incorporated, the precision is extremely important, and more accurate techniques must be used for system calibration. Also, it has been shown that the estimated statistical errors can be put into simple form by using the transformation matrix elements.

The statistical theory was verified experimentally using a simple air jet issuing at 30 m/s. For turbulence intensities of 2 and 27%, the theory correctly predicted the estimated errors for different ensemble sizes. Theoretical predictions for the estimated errors in velocity magnitudes and the velocity directions were shown to be valid.

The application of the statistical theory to 3D measurements of a high-turbulence flow with recirculation has generated meaningful vector plots. For these applications, it should be noted that accurate results can generally be obtained without using the statistical theory if large numbers of samples are collected at each location along the survey line without regard for local conditions. If, however, the statistical theory is incorporated into the data-acquisition software, the optimum number of samples will be computed as a function of local flow conditions and the specified error. In this way, test time is optimized, since additional samples are not taken where they are not required. Also, the fidelity with which each vector represents the true flow direction is constant; without the statistical theory, the directional accuracy can vary significantly along the survey line, and interpretations of the flow patterns may be more difficult.

Finally, the feasibility of following a mean streamline was considered. The experimental results indicate that the directional errors can be controlled and a smooth path obtained if the number of samples is determined from the statistical theory. In this preliminary study, the LDA test point deviated from the true streamline path. Because the deviation from the true streamline was found to be continuous in the reverse-path direction as well, it is hypothesized that an improvement in the algorithm that computes the step size and direction will result in a successful streamline-tracing technique.

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